Derivation of Planck's Constant from Maxwell's Electrodynamics

Ulrich E. Bruchholz

Schillerstrasse 36, D-04808 Wurzen, Germany

E-mail: Ulrich.Bruchholz@t-online.de; http://www.bruchholz-acoustics.de

Like Planck deduced the quantization of radiation energy from thermodynamics, the same is done from Maxwell's theory. Only condition is the existence of a geometric boundary, as deduced from author's Geometric theory of fields.

Let us go from Maxwell's equations of the vacuum that culminate in wave equations for the electric potential

$$\Box\varphi = 0 \tag{1}$$

and

$$\Box\mathcal{A} = 0 \tag{2}$$

for the magnetic vector potential.

Take the wave solution from Eqn. (2), in which the vector potential consists of a single component vertical to the propagation direction

$$A_y = A_y\Big(\omega\cdot(t-x)\Big), \tag{3}$$

where $c = 1$ (normalization), ω is a constant (identical with the circular frequency at the waves), x means the direction of the propagation, A_y is an *arbitrary* real function of $\omega\cdot(t-x)$ (independent on y, z).

The field strengthes respectively flow densities (which are the same in the vacuum) become

$$E_y = \frac{\partial A_y}{\partial t} = \omega A_y{}'\Big(\omega\cdot(t-x)\Big), \tag{4}$$

and

$$B_z = -\frac{\partial A_y}{\partial x} = \omega A_y{}'\Big(\omega\cdot(t-x)\Big), \tag{5}$$

where $A_y{}'$ means the total derivative.

The energy density of the field results in

$$\eta = \frac{\varepsilon_\circ}{2}\cdot({E_y}^2+{B_z}^2) = \omega^2\varepsilon_\circ A_y{}'^2\Big(\omega\cdot(t-x)\Big), \tag{6}$$

where $\varepsilon_\circ$ means the vacuum permitivity.

The geometric theory of fields allows geometric boundaries from the non-linearities in the equations of this theory [1]. If one assumes such a boundary, like those in stationary solutions of the non-linear equations, the included energy becomes the volume integral within this boundary

$$\iiint \eta\,\mathrm{d}(t-x)\,\mathrm{d}y\,\mathrm{d}z =$$

$$= \omega\varepsilon_\circ\iiint A_y{}'^2\Big(\omega\cdot(t-x)\Big)\,\mathrm{d}\Big(\omega\cdot(t-x)\Big)\,\mathrm{d}y\,\mathrm{d}z\,. \tag{7}$$

This volume integral would be impossible without the boundary, because the linear solution, being alone, is not physically meaningful for the infinite extension.

We can write the last equation as

$$E = \omega\,\hbar \tag{8}$$

(E means here energy), or

$$E = h\,\nu\,, \tag{9}$$

because the latter volume integral has a constant value. The known fact that this value is always the same means also that only one solution exists with ω as a parameter.

Keep the calculation for the concrete value. This can be done only in numerical way, and might be a great challenge. The value of the above volume integral has to become $\hbar/\varepsilon_\circ$. With it, the fundamental relation of Quantum Mechanics follows from classical fields.

Summarizingly, the derivation involves two predictions:

1. Photon has a geometric boundary. That may be the reason that photon behaves as a particle;
2. There is only one wave solution.

Submitted on June 23, 2009 / Accepted on July 23, 2009

References

1. Bruchholz U.E. Key notes on a geometric theory of fields. *Progress in Physics*, 2009, v. 2, 107–113.